NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2178

METHOD FOR DETERMINING OPTIMUM DIVISION OF POWER BETWEEN JET AND PROPELLER FOR MAXIMUM THRUST POWER OF A TURBINE-PROPELLER ENGINE

By Arthur M. Trout and Eldon W. Hall

Lewis Flight Propulsion Laboratory
Cleveland, Ohio

WASHINGTON
September 1950

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2178

METHOD FOR DETERMINING OPTIMUM DIVISION OF POWER BETWEEN JET AND PROPELLER FOR MAXIMUM THRUST POWER OF A TURBINE-PROPELLER ENGINE

By Arthur M. Trout and Eldon W. Hall

SUMMARY

Charts are presented by which the jet pressure ratio that gives optimum division of power between propeller and jet for maximum total thrust power may be conveniently determined for any turbine-propeller engine with or without intercooling, reheat, regeneration, or any combination of these modifications. Performance curves for various engine cycles show that the jet pressure ratio that produces maximum total power output also gives minimum or very nearly minimum specific fuel consumption.

The method of obtaining optimum power division presented herein is compared for two sets of conditions with two other methods that have been widely used for obtaining approximately optimum power division. For all conditions investigated, a simple equation (involving flight velocity, exhaust-nozzle velocity coefficient, and turbine and propeller efficiencies) for the jet velocity for maximum power from a basic turbine-propeller engine without modifications gave a value of jet velocity quite close to that calculated by using the optimum jet pressure ratio found from the charts presented. The accuracy of using the inlet-diffuser pressure ratio for the optimum jet pressure ratio depended largely on the values of component efficiencies and flight conditions used. Because the thrust-power and specific-fuel-consumption curves are relatively insensitive to a change in jet pressure ratio near the optimum, the loss in power and economy from the optimum values when the inlet-diffuser pressure ratio was used for the jet pressure ratio was less than 5 percent for the conditions investigated.

INTRODUCTION

One of the variables involved in the design or the analysis of a turbine-propeller engine is the division of power between the jet and the propeller. Although a division of power that gives maximum thrust or minimum specific fuel consumption may not necessarily be the most desirable when the load-range characteristics of an airplane powered with such an engine are considered, the general practice in the analysis of turbine-propeller-engine performance has been to use the power division giving maximum thrust as the optimum. 1361

Various methods have been employed to obtain a division of power that gives maximum thrust. A series of thrust calculations in which the jet pressure ratio is varied may be used to obtain the optimum jet pressure ratio, but this procedure requires involved calculations in order to obtain the optimum pressure ratio for each set of conditions being investigated. Several simple equations have been derived that give approximate values of optimum jet velocity for maximum thrust. One such equation (involving exhaust-nozzle velocity coefficient, turbine and propeller efficiencies, and flight velocity) is given in reference 1; this equation, however, does not consider the effects of cycle modifications such as regeneration or reheat. Moreover, determination of the turbine work output is very difficult if, in addition to the turbine-inlet conditions, only the optimum jet velocity is known instead of the optimum jet pressure ratio. Another method of determining power distribution is employed in reference 2, in which approximately optimum power distribution is obtained by taking the jet pressure ratio equal to the inlet-diffuser pressure ratio.

In order to have a direct and yet more exact method of obtaining the optimum jet pressure ratio for use in calculating the performance of various turbine-propeller-engine cycles and to check the accuracy of the simpler methods, an analysis was made at the NACA Lewis laboratory to obtain an expression for the optimum jet pressure ratio for any turbine-propeller engine. The results of this analysis are presented in the form of charts from which the jet pressure ratio for the division of power giving maximum thrust may be obtained for turbine-propeller engines either with or without intercooling, reheat, regeneration, or any combination of these modifications.

DESCRIPTION OF ENGINE CYCLE

A schematic diagram of a turbine-propeller engine incorporating all the modifications considered in the analysis is presented in figure 1. Engine components for the basic cycle are represented by solid lines; additional equipment necessary for the various

1361

cycle modifications is shown by dashed lines. Air enters the engine and passes through the inlet diffuser, the first compressor, and into the intercooler. In the intercooler, some of the heat from the partly compressed air is given up to the air passing through the cold side of the intercooler; the cooled air then flows into the second compressor where it is further compressed. From the second compressor, the air passes through the cold side of the regenerator and receives heat from the gases passing through the hot side of the regenerator; the air then passes into the primary burner where fuel is added and burned. The hot gases from the burner expand through the first turbine and enter the reheat burner, where more fuel is added and burned. After expanding through the second turbine, the gases pass through the hot side of the regenerator, where they give up heat to the air passing through the cold side; the gases then leave the engine through the jet nozzle. The turbine furnishes the power required by the compressor and also supplies power to the propeller through reduction gearing.

ANALYSIS

Development of Charts for Determining Optimum Jet Pressure Ratio

All symbols used in the derivation of the charts are defined in appendix A. An equation is derived in appendix B that expresses the total thrust per unit gas flow for a turbine-propeller engine with regeneration and intercooling in terms of operating and design constants and the jet pressure ratio. This equation is then differentiated with respect to the jet pressure ratio and set equal to zero; a general expression containing various operating and design constants and the jet pressure ratio that gives maximum thrust is thereby obtained. When this expression is rearranged, the following equation is obtained comprising the parameter A, which is a function of the jet pressure ratio for maximum thrust, and the two parameters B and C, which contain operating and design constants:

$$B = \frac{A^3(A-1)}{C}\left(1 - \frac{C}{A^2} + \sqrt{1 - \frac{2C}{A^2}}\right) - A \qquad \text{(B29)}$$

where

1361

$$A = \left(\frac{P_6}{p_0}\right)_{opt}^{\frac{\gamma_t - 1}{\gamma_t}}$$

$$B = \left(K \frac{P_2}{p_0}\right)^{\frac{\gamma_t - 1}{\gamma_t}} \left[\frac{1}{\eta_t}\left(1 + \frac{\eta_x'}{1-\eta_x'} \frac{T_2}{T_4}\right) - 1\right]$$

and

$$C = \left(K \frac{P_2}{p_0}\right)^{\frac{\gamma_t - 1}{\gamma_t}} \left(\frac{C_v}{\eta_p}\right)^2 \frac{1-\eta_x'}{\eta_t} \frac{V_0^2}{Jgc_{p,t}T_4}$$

Because equation (B29) cannot be readily solved for the optimum jet pressure ratio, it has been plotted in figure 2(a) for a wide range of values of the parameters A, B, and C. Thus the jet pressure ratio giving maximum thrust for any turbine-propeller engine without reheat and with or without intercooling, regeneration, or both may be determined from figure 2(a) by evaluating the two parameters B and C.

In appendix C, the following expression similar to that derived in appendix B is derived for an engine with two turbines and reheating between the turbines with intercooling, regeneration, or both:

$$B_r = \frac{A^2(A-1)}{C_r}\left(1 - \frac{C_r}{2A^{3/2}} + \sqrt{1 - \frac{C_r}{A^{3/2}}}\right) - \sqrt{A} \qquad \text{(C7)}$$

where

$$A = \left(\frac{P_6}{p_0}\right)_{opt}^{\frac{\gamma_t - 1}{\gamma_t}}$$

$$B_r = \left(K\frac{P_2}{p_0}\right)^{\frac{\gamma_t-1}{2\gamma_t}}\left[\frac{1}{\eta_t}\left(1+\frac{\eta_x'}{1-\eta_x'}\frac{T_2}{T_4}\right)-1\right]$$

and

$$C_r = \left(K\frac{P_2}{p_0}\right)^{\frac{\gamma_t-1}{2\gamma_t}}\left(\frac{C_v}{\eta_p}\right)^2\frac{1-\eta_x'}{\eta_t}\frac{V_0^2}{Jgc_{p,t}T_4}$$

In this derivation, the pressure ratio in each turbine was assumed equal to the square root of the over-all effective turbine pressure ratio $(P_4/P_5)_e$, and the temperature after reheating was assumed equal to the inlet temperature of the first turbine. The validity of these assumptions is discussed in appendix D. Equation (C7) is plotted in figure 2(b) for a wide range of values of the parameters A, B_r, and C_r. The jet pressure ratio giving maximum thrust for any turbine-propeller engine with reheating as previously described and either with or without intercooling, regeneration, or both may be determined from figure 2(b) by evaluating the two parameters B_r and C_r.

If the engine being considered does not have regeneration, the value of η_x' in parameters B and C in figure 2(a) and in parameters B_r and C_r in figure 2(b) is taken as zero. For an engine with intercooling in combination with regeneration, a value of T_2 must be calculated for use in parameter B of figure 2(a) or B_r of figure 2(b). If the engine does not have regeneration, however, the term involving T_2 drops out and the intercooling has no effect upon the optimum jet pressure ratio, except as the pressure loss in the intercooler or an increase in compressor pressure ratio may affect the term $\left(K\frac{P_2}{p_0}\right)^{\frac{\gamma_t-1}{\gamma_t}}$

When the flight speed V_0 is zero, the propeller efficiency η_p is also zero and parameters C (fig. 2(a)) and C_r (fig. 2(b)) are indeterminate. For this condition and also for very low flight speeds (such as take-off conditions) where propeller efficiency may be of questionable accuracy, V_0/η_p should be replaced by the expression $550/\varphi$ where φ is the pounds of propeller thrust per shaft horsepower input.

1361

Other Methods of Power Division

The following approximate equation is derived in reference 1 for the jet velocity that will give maximum thrust for a basic turbine-propeller engine without modifications if propeller and turbine efficiencies are constant:

$$V_{j,opt} = \frac{V_0 \, C_v^2}{\eta_p \, \eta_t'} \tag{1}$$

where η_t' is the turbine polytropic efficiency. If η_p and η_t' are not constant with change in jet velocity, differential efficiencies must be used as explained in reference 1.

In reference 2, the jet pressure ratio is considered equal to the inlet-diffuser pressure ratio as a means of determining power distribution for the basic turbine-propeller engine and engines with intercooling, reheating, and regeneration; that is

$$\frac{p_6}{p_0} = \frac{p_1}{p_0} \tag{2}$$

DISCUSSION

In order to compare the method presented herein with the other methods of obtaining power division and to show the range of values of jet pressure ratio and ratio of jet to propeller thrust that are generally encountered, calculations were made of the performance of turbine-propeller engines having various cycle modifications for two conditions of component efficiencies and altitude and for a range of flight velocities. The values chosen for the calculations are as follows:

	Condition I	Condition II
Propeller efficiency (including reduction gearing), η_p	0.80	0.95
Inlet-diffuser efficiency, η_d	0.85	0.95
Compressor adiabatic efficiency without intercooling, η_c	0.80	0.90
Compressor adiabatic efficiency for each compressor with intercooling, η_c	0.825	0.913
Turbine adiabatic efficiency without reheat, η_t	0.85	0.95
Turbine adiabatic efficiency for each turbine with reheat, η_t	0.833	0.943
Exhaust-nozzle velocity coefficient, C_v	0.975	0.975
Compressor pressure ratio, P_2/P_1	8	8
Turbine-inlet temperature, T_4 (also T_{5r} with reheat), ^{o}R	2000	2000
Intercooler effectiveness, η_i	0.50	0.50
Regenerator heating effectiveness, η_x	0.70	0.70
Altitude, ft	0	35,300

Values of component efficiencies and altitude for condition II higher than for condition I were selected in order to indicate the greatest differences between the various methods that would usually be encountered.

In the cycles with intercooling, intercooling was assumed to occur at the square root of the over-all compressor pressure ratio. The compressor adiabatic efficiency was higher with intercooling than without, because an adiabatic efficiency was calculated for each compressor such that the change in enthalpy for the two compressors with zero intercooling effectiveness would be equal to the change in enthalpy for the same pressure ratio with a single compressor. A similar consideration was made for the turbine efficiency for the reheat cycles. The following values of the pressure-loss factor K were chosen for both sets of conditions:

	K
Basic cycle and intercooling cycle	0.97
Reheat cycle	.92
Regenerative cycles	.89

1361

For the cycles with intercooling, an intercooler pressure loss of $0.045\ P_{1i}$ was used.

The thrust horsepower per unit gas flow and specific fuel consumption for turbine-propeller engines representative of various engine cycles are shown in figure 3 for a range of jet pressure ratios and a flight velocity of 500 miles per hour (733 ft/sec) for conditions I and II. The jet pressure ratios were chosen to give jet velocities from approximately $\frac{1}{2}$ to $1\frac{1}{2}$ times the jet velocity for maximum thrust power for each engine cycle. As the jet pressure ratio is increased from the lowest value shown, the thrust horsepower increases to a maximum and then decreases as the jet pressure ratio is further increased. The jet pressure ratio obtained from figure 2 is indicated on each of the curves in figure 3 and occurs at the maximum thrust power for each of the cycles. For each set of conditions, the optimum jet pressure ratio for maximum thrust power for all the cycles considered is about the same; it occurs in a range of 1.24 to 1.33 for condition I (fig. 3(a)) and 1.12 to 1.18 for condition II (fig. 3(b)). A point corresponding to the jet velocity given by equation (1) appears on the curves for the basic engine cycle and for the intercooling cycle as well, inasmuch as intercooling without regeneration has no effect upon the optimum jet velocity. In evaluating the jet velocity given by equation (1), the polytropic turbine efficiency η_t' corresponding to the assumed adiabatic efficiency was used. The jet pressure ratio corresponding to the jet velocity given by equation (1) is in good agreement with the optimum jet pressure ratios from figure 2. The jet pressure ratio equal to the inlet-diffuser pressure ratio (equation (2)) is also shown on each of the curves in figure 3. For condition I, the inlet-diffuser pressure ratio (1.28) falls quite close to the optimum jet pressure ratio for the various cycles, but for condition II the inlet-diffuser pressure ratio (1.43) is considerably higher than the optimum jet pressure ratio for all the engine cycles shown. The thrust horsepower, however, does not vary markedly with a change in jet pressure ratio in this region; consequently, the decrease in power at the point given by equation (2) is less than 5 percent from the maximum value for all cycles and conditions investigated.

1361

The inlet-diffuser pressure ratio can be changed only by a change in flight conditions or inlet-diffuser efficiency; whereas the optimum jet pressure ratio may be changed either by variations in the flight conditions or by variations in engine component efficiencies and engine operating conditions, as may be seen by inspection of the parameters B and C in figure 2. For a given flight velocity, a change in altitude has very little effect on either the optimum jet pressure ratio found from figure 2 or the jet pressure ratio corresponding to the jet velocity given by equation (1). The jet pressure ratio equal to the inlet-diffuser pressure ratio (equation (2)) will change, however, because of the change in diffuser inlet temperature. The increase in altitude from condition I to condition II accounts for a large part of the increase in jet pressure ratio, as obtained from equation (2). The decrease in optimum jet pressure ratio as determined by figure 2 is a result of the higher component efficiencies. For the efficiencies of condition II, but at sea-level altitude, the differences between the jet pressure ratio as obtained from equation (2) and the optimum jet pressure ratio as found from figure 2 would be approximately one-half as great as for an altitude of 35,300 feet.

N

The specific-fuel-consumption curves in figure 3 for the basic cycle and for the cycle with intercooling show that minimum specific fuel consumption for these cycles occurs at the same jet pressure ratio that gives maximum thrust. This coincidence of optimum jet pressure ratio for maximum thrust and minimum specific fuel consumption occurs because the fuel flow required by turbine-propeller engines either with or without intercooling is independent of the jet pressure ratio for a fixed set of operating conditions. For engine cycles with regeneration or reheat, however, a change in jet pressure ratio while constant operating conditions are maintained causes not only a change in turbine power output and consequently turbine-outlet temperature but also a change in the fuel rate required to maintain a fixed turbine-inlet temperature (or to maintain a fixed temperature at the inlet to each turbine for the engine with reheating). Because the decrease in total thrust that accompanies a change in the jet pressure ratio is very small near the maximum output, a range exists in which an increase in jet pressure ratio beyond the optimum for maximum thrust causes a decrease in the fuel required that is proportionately larger than the loss in total thrust; a lower specific fuel consumption therefore results. This principle is illustrated by the specific-fuel-consumption curves for the engine cycles with reheat or regeneration in figure 3, which show that minimum specific fuel consumption for these cycles occurs at a slightly higher jet pressure ratio than the pressure ratio that gives maximum thrust power. The minimum specific fuel consumption for these cycles, however, is only about

1 percent lower than the specific fuel consumption at maximum thrust for the chosen conditions. Calculation of the jet pressure ratio that would give minimum specific fuel consumption would be considerably more complicated than that for maximum thrust; this calculation is unwarranted in view of the small difference in specific fuel consumption between maximum thrust and maximum fuel economy for engines with regeneration or reheat.

The ratio of jet thrust to propeller thrust for optimum power division is plotted in figure 4 against the square of flight velocity for the same engine cycles and operating conditions that were used in figure 3. Also shown in figure 4 are curves of optimum jet pressure ratio for the basic and reheat cycles of figure 3 along with the jet pressure ratio for the basic engine that corresponds to the jet velocity of equation (1) and the jet pressure ratio obtained from equation (2) (inlet-diffuser pressure ratio); all the curves are plotted against the square of flight velocity. The curves for the optimum jet pressure ratio of the other cycles of figure 3 all lie between the basic- and reheat-cycle values and were omitted for simplicity.

Increasing the flight velocity increases the optimum jet pressure ratio and the ratio of jet thrust to propeller thrust approximately in proportion to the square of the flight velocity (fig. 4). For condition I, the amount of thrust obtained from the jet with optimum power distribution is small in comparison with the thrust obtained from the propeller at lower flight velocities, but increases to as much as 50 percent for the basic cycle at about 600 miles per hour. For condition II, the ratio of jet thrust to propeller thrust at optimum power division is very close to zero for all cycles and flight velocities. At 600 miles per hour for the basic engine cycle, the ratio of jet thrust to propeller thrust is only 0.03, and for the engine cycles with regeneration, the ratio of jet thrust to propeller thrust is negative for all flight velocities, which indicates that optimum jet velocity is lower than flight velocity for these cycles. (In fig. 4(b), the basic- and intercooling-cycle curves for the thrust ratio coincide.)

The curve of the jet pressure ratio corresponding to the jet velocity given by equation (1) is in good agreement with the optimum jet pressure ratio for the basic cycle obtained from figure 2(a) over the range of flight velocities considered for both sets of operating conditions. Calculations for various component efficiencies, compressor pressure ratios, and turbine-inlet temperatures showed that equation (1) consistently gave jet velocities in good agreement with the jet velocity for the basic engine cycle obtained by using

1361

the optimum jet pressure ratio from figure 2(a). For condition I, the curve representing equation (2) falls about midway between the optimum jet pressure ratios for the basic and reheat cycles for the range of flight velocities shown; for condition II, however, this curve is much higher than the optimum jet pressure ratios for all engine cycles at all flight velocities.

A constant propeller efficiency was assumed at all flight velocities in figure 4. If propeller efficiency decreased with increasing flight velocity, the curves for optimum jet pressure ratio and the curve corresponding to the jet velocity given by equation (1) would be displaced toward higher values; the good agreement between the optimum jet pressure ratio for the basic cycle and the pressure ratio corresponding to equation (1), however, would not be changed. The curve representing equation (2) (the inlet-diffuser pressure ratio) would, of course, remain the same.

For condition II, the compressor pressure ratio and the turbine-inlet temperature were assumed to be the same as for condition I. Additional calculations showed that increasing the compressor pressure ratio increases the optimum jet pressure ratio slightly, whereas increasing the turbine-inlet temperature decreases the optimum jet pressure ratio slightly; therefore, an increase in both compressor pressure ratio and turbine-inlet temperature would result in about the same optimum jet pressure ratios as in figure 4.

SUMMARY OF RESULTS

Charts are presented herein that make possible, with a minimum of calculations, the determination of the jet pressure ratio that gives maximum thrust power for any given turbine-propeller engine with or without intercooling, reheat, or regeneration or any combination of these modifications.

A value of jet velocity very close to the optimum jet velocity for the basic engine cycle without modifications and for the cycle with intercooling was found by means of a simple equation involving flight velocity, exhaust-nozzle velocity coefficient, and turbine and propeller efficiencies.

The accuracy of using the inlet-diffuser pressure ratio for the jet pressure ratio to obtain approximately optimum power distribution for any turbine-propeller-engine cycle depended largely upon the values of component efficiencies and flight conditions used; however, because both thrust power and specific fuel consumption are relatively insensitive to changes in jet pressure ratio near the optimum, the loss in power and economy from the maximum values, when this approximate method of power distribution was used, was less than 5 percent for the conditions investigated.

Lewis Flight Propulsion Laboratory,
National Advisory Committee for Aeronautics,
Cleveland, Ohio, March 9, 1950.

APPENDIX A

SYMBOLS

The following symbols are used in this report:

A, B, B_r, C, and C_r	parameters used in plotting figure 2
C_V	exhaust-nozzle velocity coefficient
c_p	specific heat at constant pressure, Btu/(lb)(°R)
F	thrust, lb
f	specific fuel consumption, lb/thp-hr
f/a	fuel-air ratio
g	acceleration due to gravity, 32.17 ft/sec^2
J	mechanical equivalent of heat, 778.3 ft-lb/Btu
K	factor accounting for pressure losses, $\left(1 - \frac{\Delta P_{xc}}{P_2}\right)\left(1 - \frac{\Delta P_b}{P_3}\right)\left(1 - \frac{\Delta P_r}{P_{4r}}\right)\left(1 - \frac{\Delta P_{xh}}{P_5}\right)$
P	total pressure, lb/sq in. absolute
$(P_4/P_5)_e$	effective turbine pressure ratio, (equal to $(P_4/P_{4r})(P_{5r}/P_5)$ with reheat burner or P_4/P_5 without reheat burner)
$(P_6/p_0)_{opt}$	optimum jet pressure ratio for maximum total thrust
ΔP	total pressure loss, lb/sq in.
p	static pressure, lb/sq in. absolute
shp	shaft horsepower
T	total temperature, °R
ΔT	change in total temperature, °R

1361

t	static temperature, °R
thp	thrust horsepower
V_0	flight velocity, ft/sec
V_j	jet velocity relative to engine, ft/sec
W	mass flow of air, lb/sec
γ	ratio of specific heats
η_c	compressor adiabatic efficiency without intercooling (for intercooling, each compressor considered separately), $\dfrac{T_1\left[\left(\dfrac{P_2}{P_1}\right)^{\frac{\gamma_c-1}{\gamma_c}} - 1\right]}{T_2 - T_1}$
η_d	inlet-diffuser efficiency, $\dfrac{t_0\left[\left(\dfrac{P_1}{p_0}\right)^{\frac{\gamma_d-1}{\gamma_d}} - 1\right]}{T_1 - t_0}$
η_i	intercooler effectiveness, $\dfrac{T_{1i} - T_{2i}}{T_{1i} - T_1}$
η_p	propeller efficiency (also accounts for reduction gear losses), $\dfrac{F_p V_0}{550 \text{ shp}}$

η_t turbine adiabatic efficiency without reheat, (for reheat, each turbine considered separately),

$$\frac{T_4 - T_5}{T_4\left[1 - \left(\frac{P_5}{P_4}\right)^{\frac{\gamma_t - 1}{\gamma_t}}\right]}$$

η_x regenerator heating effectiveness, $\frac{T_3-T_2}{T_5-T_2}$

η_x' regenerator cooling effectiveness, $\frac{c_{p,xc}}{c_{p,xh}(1+f/a)}\ \eta_x$

φ propeller thrust per shaft power input, lb/hp

Subscripts:

b primary combustion chamber

c compressor

d inlet diffuser

e effective

i intercooler

j jet

opt optimum for maximum thrust power

p propeller

r reheat burner or with reheating

t turbine

xc regenerator cold side

xh regenerator hot side

Numbered subscripts refer to stations shown in figure 1.

0	ambient air
1	compressor inlet
1i	first-compressor outlet or intercooler inlet
2	second-compressor outlet or regenerator cold-side inlet
2i	intercooler outlet or second-compressor inlet
3	regenerator cold-side outlet or combustion-chamber inlet
4	combustion-chamber outlet or turbine inlet
4r	first-turbine outlet or reheat-burner inlet
5	turbine outlet or regenerator hot-side inlet
5r	reheat-burner outlet or second-turbine inlet
6	regenerator hot-side outlet or jet-nozzle inlet

1361

1361

APPENDIX B

DERIVATION OF EXPRESSION FOR OPTIMUM JET PRESSURE RATIO FOR ENGINE WITH OR WITHOUT INTERCOOLING, REGENERATION, OR BOTH

For the derivation of optimum jet pressure ratio for a turbine-propeller engine, including the effects of intercooling and regeneration, the following operating and design constants are considered: C_v, $c_{p,c}$, $c_{p,t}$, P_1, P_2, P_4, p_0, T_1, T_2, T_4, V_0, W, γ_t, η_t, η_p, and η_x'. The variables are: F, P_5, P_6, shp_p, T_3, T_5, T_6, and V_j.

Among the following equations, those equations that contain η_x or η_x' may be applied to turbine-propeller engines without regeneration if the values of η_x and η_x' are considered zero.

The net thrust of the jet, including the effect of fuel burned, is

$$F_j = \frac{W}{g}\left[(1+f/a)\, V_j - V_0\right] \tag{B1}$$

and the propeller thrust is

$$F_p = \frac{550\, \eta_p\, shp_p}{V_0} \tag{B2}$$

where the propeller efficiency η_p also accounts for the reduction-gear losses.

The total thrust per unit gas flow is therefore

$$\frac{F}{\frac{W}{g}(1+f/a)} = V_j - \frac{V_0}{1+f/a} + \frac{550\, \eta_p\, shp_p}{\frac{W}{g}\, V_0(1+f/a)} \tag{B3}$$

If it is assumed that γ and c_p are the same for turbine and exhaust gases (appendix D), the jet velocity is given by

$$V_j = \sqrt{C_v^2 2Jgc_{p,t}T_6 \left[1 - \left(\frac{p_0}{P_6}\right)^{\frac{\gamma_t - 1}{\gamma_t}}\right]} \tag{B4}$$

1361

By definition,

$$\eta_x = \frac{T_3 - T_2}{T_5 - T_2} \tag{B5}$$

Also, the heat gained by the compressed air in the regenerator is equal to the heat lost by the exhaust gases so that

$$Wc_{p,xc}(T_3 - T_2) = W(1+f/a)c_{p,xh}(T_5 - T_6) \tag{B6}$$

From equations (B5) and (B6),

$$\frac{c_{p,xc}}{c_{p,xh}(1+f/a)}\eta_x = \frac{T_5 - T_6}{T_5 - T_2} \tag{B7}$$

Letting,

$$\eta_x' = \frac{c_{p,xc}}{c_{p,xh}(1+f/a)}\eta_x \tag{B8}$$

then

$$T_6 = T_5 - \eta_x'(T_5 - T_2) = T_5(1 - \eta_x') + \eta_x' T_2 \tag{B9}$$

The turbine-outlet temperature is

$$T_5 = T_4\left\{1 - \eta_t\left[1 - \left(\frac{P_5}{P_4}\right)_e^{\frac{\gamma_t - 1}{\gamma_t}}\right]\right\} \tag{B10}$$

The term $\left(\frac{P_5}{P_4}\right)_e$ is introduced at this point to render the subsequent equations applicable to pressure losses in an engine having a two-stage turbine with reheating between stages (appendix C). The effective turbine pressure ratio $\left(\frac{P_4}{P_5}\right)_e$ is equal to $\frac{P_4}{P_{4r}}\,\frac{P_{5r}}{P_5}$ with reheating or $\frac{P_4}{P_5}$ without reheating.

1361

When equations (B4), (B9), and (B10) are combined,

$$V_j = \sqrt{C_V^{\,2} 2Jgc_{p,t}\left[1 - \left(\frac{P_6}{P_0}\right)^{-\frac{\gamma_t-1}{\gamma_t}}\right]\left((1-\eta_x')\,T_4\left\{1 - \eta_t\left[1 - \left(\frac{P_5}{P_4}\right)_e^{\frac{\gamma_t-1}{\gamma_t}}\right]\right\} + \eta_x'\,T_2\right)} \qquad \text{(B11)}$$

The turbine power per unit gas flow is

$$\frac{shp_t}{\frac{W}{g}(1+f/a)} = \frac{\eta_t J g c_{p,t} T_4}{550}\left[1 - \left(\frac{P_5}{P_4}\right)_e^{\frac{\gamma_t-1}{\gamma_t}}\right] \qquad \text{(B12)}$$

and the compressor power per unit air flow is

$$\frac{shp_c}{\frac{W}{g}} = \frac{Jgc_{p,c}\,\Delta T_c}{550} \qquad \text{(B13)}$$

where, without intercooling,

$$\Delta T_c = T_2 - T_1$$

or with intercooling,

$$\Delta T_c = (T_{1i} - T_1) + (T_2 - T_{2i})$$

The shaft power available to the propeller is the difference between turbine and compressor power; thus,

$$\frac{shp_p}{\frac{W}{g}(1+f/a)} = \frac{\eta_t J g c_{p,t} T_4}{550}\left[1 - \left(\frac{P_5}{P_4}\right)_e^{\frac{\gamma_t - 1}{\gamma_t}}\right] - \frac{J g c_{p,c}\,\Delta T_c}{550(1+f/a)} \tag{B14}$$

The propeller thrust per unit gas flow is

$$\frac{F_p}{\frac{W}{g}(1+f/a)} = \frac{550\eta_p shp_p}{\frac{W}{g}V_0(1+f/a)} = \frac{\eta_p \eta_t J g c_{p,t} T_4}{V_0}\left[1 - \left(\frac{P_5}{P_4}\right)_e^{\frac{\gamma_t - 1}{\gamma_t}}\right] - \frac{\eta_p J g c_{p,c}\,\Delta T_c}{V_0(1+f/a)} \tag{B15}$$

By definition,

$$\left(\frac{P_4}{P_5}\right)_e = K\frac{P_2}{P_6} \tag{B16}$$

or

$$\left(\frac{P_5}{P_4}\right)_e = \frac{\frac{P_6}{p_0}}{K\frac{P_2}{p_0}} \tag{B17}$$

Because,

$$\left(\frac{P_4}{P_5}\right)_e = \frac{P_4}{P_{4r}}\frac{P_{5r}}{P_5} = \frac{P_4}{P_5}\frac{P_{5r}}{P_{4r}} \tag{B18}$$

the value of K from equation (B16) becomes

$$K = \frac{P_4}{P_5}\frac{P_{5r}}{P_{4r}}\frac{P_6}{P_2} = \frac{P_4}{P_2}\frac{P_{5r}}{P_{4r}}\frac{P_6}{P_5} \tag{B19}$$

$$K = \frac{P_3}{P_2}\frac{P_4}{P_3}\frac{P_{5r}}{P_{4r}}\frac{P_6}{P_5} \tag{B20}$$

When K is expressed in terms of the total-pressure losses in the cold side of the regenerator, the primary combustion chamber, the reheat combustion chamber, and the hot side of the regenerator, from equation (B20),

$$K = \left(1 - \frac{\Delta P_{xc}}{P_2}\right)\left(1 - \frac{\Delta P_b}{P_3}\right)\left(1 - \frac{\Delta P_r}{P_{4r}}\right)\left(1 - \frac{\Delta P_{xh}}{P_5}\right) \tag{B21}$$

If equation (B17) is used, the jet velocity and the propeller thrust may be expressed in terms of the primary variable P_6/p_0 and operating and design constants. From equations (B11) and (B17),

$$V_j = \sqrt{c_v^2 2Jgc_{p,t}\left[1 - \left(\frac{P_6}{p_0}\right)^{-\frac{\gamma_t-1}{\gamma_t}}\right]\left((1-\eta_x')T_4\left\{1 - \eta_t\left[1 - \left(K\frac{P_2}{p_0}\right)^{-\frac{\gamma_t-1}{\gamma_t}}\left(\frac{P_6}{p_0}\right)^{\frac{\gamma_t-1}{\gamma_t}}\right]\right\} + \eta_x' T_2\right)} \tag{B22}$$

From equations (B15) and (B17), the propeller thrust per unit gas flow is

$$\frac{F_p}{\frac{W}{g}(1+f/a)} = \frac{550\eta_p shp_p}{\frac{W}{g}V_0(1+f/a)} = \frac{\eta_p\eta_t Jgc_{p,t}T_4}{V_0}\left[1 - \left(K\frac{P_2}{p_0}\right)^{-\frac{\gamma_t-1}{\gamma_t}}\left(\frac{P_6}{p_0}\right)^{\frac{\gamma_t-1}{\gamma_t}}\right] - \frac{\eta_p Jgc_{p,c}\,\Delta T_c}{V_0(1+f/a)} \tag{B23}$$

When equations (B22) and (B23) are substituted in equation (B3), the total thrust per unit gas flow in terms of the jet pressure ratio and operating and design constants is

$$\frac{F}{\frac{W}{g}(1+f/a)} = \sqrt{c_v^{\,2}2Jgc_{p,t}\left[1-\left(\frac{P_6}{p_0}\right)^{-\frac{\gamma_t-1}{\gamma_t}}\right]\left((1-\eta_x')T_4\left\{1-\eta_t\left[1-\left(K\frac{P_2}{p_0}\right)^{-\frac{\gamma_t-1}{\gamma_t}}\left(\frac{P_6}{p_0}\right)^{\frac{\gamma_t-1}{\gamma_t}}\right]\right\}+\eta_x'T_2\right)} -$$

$$\frac{V_0}{1+f/a} + \frac{\eta_p\eta_t Jgc_{p,t}T_4}{V_0}\left[1-\left(K\frac{P_2}{p_0}\right)^{-\frac{\gamma_t-1}{\gamma_t}}\left(\frac{P_6}{p_0}\right)^{\frac{\gamma_t-1}{\gamma_t}}\right] - \frac{\eta_p Jgc_{p,c}\,\Delta T_c}{V_0(1+f/a)}$$

(B24)

In order to find the value of $\left(\frac{P_6}{p_0}\right)^{\frac{\gamma_t-1}{\gamma_t}}$ that will give maximum thrust, the derivative of the total thrust with respect to $\left(\frac{P_6}{p_0}\right)^{\frac{\gamma_t-1}{\gamma_t}}$ is set equal to zero.

$$\frac{d\,\dfrac{F}{\frac{W}{g}(1+f/a)}}{d\left(\dfrac{P_6}{p_0}\right)^{\frac{\gamma_t-1}{\gamma_t}}} = 0 \qquad \text{(B25)}$$

From equations (B24) and (B25),

$$\frac{C_v{}^2 2Jgc_{p,t}\left((1-\eta_x')T_4\left\{1-\eta_t\left[1-\left(K\frac{p_2}{p_0}\right)^{-\frac{\gamma_t-1}{\gamma_t}}\left(\frac{p_6}{p_0}\right)_{opt}^{-\frac{\gamma_t-1}{\gamma_t}}\right]\right\}+\eta_x'T_8\right)\left(\frac{p_6}{p_0}\right)_{opt}^{-2\frac{\gamma_t-1}{\gamma_t}}+C_v{}^2 2Jgc_{p,t}\left[1-\left(\frac{p_6}{p_0}\right)_{opt}^{-\frac{\gamma_t-1}{\gamma_t}}\right](1-\eta_x')T_4\,\eta_t\left(K\frac{p_2}{p_0}\right)^{-\frac{\gamma_t-1}{\gamma_t}}}{2\sqrt{C_v{}^2 2Jgc_{p,t}\left[1-\left(\frac{p_6}{p_0}\right)_{opt}^{-\frac{\gamma_t-1}{\gamma_t}}\right]\left((1-\eta_x')T_4\left\{1-\eta_t\left[1-\left(K\frac{p_2}{p_0}\right)^{-\frac{\gamma_t-1}{\gamma_t}}\left(\frac{p_6}{p_0}\right)_{opt}^{\frac{\gamma_t-1}{\gamma_t}}\right]\right\}+\eta_x'T_8\right)}}=\frac{\eta_p\eta_t Jgc_{p,t}T_4}{V_0}\left(K\frac{p_2}{p_0}\right)^{-\frac{\gamma_t-1}{\gamma_t}}$$

(B26)

Then

$$\left[\frac{1}{2\left(\frac{P_6}{P_0}\right)_{opt}^{2\frac{\gamma_t-1}{\gamma_t}}}\right]\sqrt{\frac{(1-\eta_x')T_4\left\{1-\eta_t\left[1-\left(K\frac{P_2}{P_0}\right)^{-\frac{\gamma_t-1}{\gamma_t}}\left(\frac{P_6}{P_0}\right)_{opt}^{\frac{\gamma_t-1}{\gamma_t}}\right]\right\}+\eta_x' T_2}{1-\left(\frac{P_6}{P_0}\right)_{opt}^{-\frac{\gamma_t-1}{\gamma_t}}}}\ +$$

$$\left[\frac{1}{2}(1-\eta_x')T_4\,\eta_t\left(K\frac{P_2}{P_0}\right)^{-\frac{\gamma_t-1}{\gamma_t}}\right]\sqrt{\frac{1-\left(\frac{P_6}{P_0}\right)_{opt}^{-\frac{\gamma_t-1}{\gamma_t}}}{(1-\eta_x')T_4\left\{1-\eta_t\left[1-\left(K\frac{P_2}{P_0}\right)^{-\frac{\gamma_t-1}{\gamma_t}}\left(\frac{P_6}{P_0}\right)_{opt}^{\frac{\gamma_t-1}{\gamma_t}}\right]\right\}+\eta_x' T_2}}$$

$$=\frac{\eta_p\eta_t J g c_{p,t} T_4}{V_0\sqrt{C_v{}^2 2Jgc_{p,t}}}\left(K\frac{P_2}{P_0}\right)^{-\frac{\gamma_t-1}{\gamma_t}}$$

(B27)

and

$$\sqrt{2\left(K\frac{p_2}{p_0}\right)^{\frac{\gamma_t-1}{\gamma_t}}\left(\frac{C_v}{\eta_p}\right)^2\frac{1-\eta_x'}{\eta_t}\frac{V_0^2}{Jgc_{p,t}T_4}\frac{\left(K\frac{p_2}{p_0}\right)^{\frac{\gamma_t-1}{\gamma_t}}\left[\frac{1}{\eta_t}\left(1+\frac{\eta_x'}{1-\eta_x'}\frac{T_2}{T_4}\right)-1\right]+\left(\frac{p_6}{p_0}\right)_{opt}^{\frac{\gamma_t-1}{\gamma_t}}}{\left(\frac{p_6}{p_0}\right)_{opt}^{4\frac{\gamma_t-1}{\gamma_t}}-\left(\frac{p_6}{p_0}\right)_{opt}^{3\frac{\gamma_t-1}{\gamma_t}}}}+$$

$$\sqrt{2\left(K\frac{p_2}{p_0}\right)^{\frac{\gamma_t-1}{\gamma_t}}\left(\frac{C_v}{\eta_p}\right)^2\frac{1-\eta_x'}{\eta_t}\frac{V_0^2}{Jgc_{p,t}T_4}\frac{1-\left(\frac{p_6}{p_0}\right)_{opt}^{-\frac{\gamma_t-1}{\gamma_t}}}{\left(K\frac{p_2}{p_0}\right)^{\frac{\gamma_t-1}{\gamma_t}}\left[\frac{1}{\eta_t}\left(1+\frac{\eta_x'}{1-\eta_x'}\frac{T_2}{T_4}\right)-1\right]+\left(\frac{p_6}{p_0}\right)_{opt}^{\frac{\gamma_t-1}{\gamma_t}}}}=2$$

(B28)

Letting

$$A=\left(\frac{p_6}{p_0}\right)_{opt}^{\frac{\gamma_t-1}{\gamma_t}}$$

$$B=\left(K\frac{p_2}{p_0}\right)^{\frac{\gamma_t-1}{\gamma_t}}\left[\frac{1}{\eta_t}\left(1+\frac{\eta_x'}{1-\eta_x'}\frac{T_2}{T_4}\right)-1\right]$$

and

$$C=\left(K\frac{p_2}{p_0}\right)^{\frac{\gamma_t-1}{\gamma_t}}\left(\frac{C_v}{\eta_p}\right)^2\frac{1-\eta_x'}{\eta_t}\frac{V_0^2}{Jgc_{p,t}T_4}$$

1361

equation (B28) may be written

$$\sqrt{\frac{2C(B+A)}{A^3(A-1)}} + \sqrt{\frac{2C(A-1)}{A(B+A)}} = 2 \qquad \text{(B29)}$$

or

$$\sqrt{\frac{2C}{A^3(A-1)}}\left(\sqrt{B+A}\right)^2 - 2\sqrt{B+A} + \sqrt{\frac{2C(A-1)}{A}} = 0 \qquad \text{(B30)}$$

When equation (B30) is solved for $\sqrt{B+A}$

$$\sqrt{B+A} = \frac{1 \pm \sqrt{1 - \sqrt{\frac{2C}{A^3(A-1)}}\sqrt{\frac{2C(A-1)}{A}}}}{\sqrt{\frac{2C}{A^3(A-1)}}} \qquad \text{(B31)}$$

(Upon examination, the positive value was found to give the desired root.) and

$$B = \frac{A^3(A-1)}{C}\left(1 - \frac{C}{A^2} + \sqrt{1 - \frac{2C}{A^2}}\right) - A \qquad \text{(B32)}$$

Equation (B32) is plotted in figure 2(a).

1361

APPENDIX C

DERIVATION OF EXPRESSION FOR OPTIMUM JET PRESSURE RATIO

FOR ENGINE WITH REHEAT, WITH OR WITHOUT INTERCOOLING,

REGENERATION, OR BOTH

If it is assumed that reheat occurs at the square root of the over-all turbine pressure ratio and that the temperature after reheating is equal to the turbine-inlet temperature,

$$\frac{P_{5r}}{P_5} = \frac{P_4}{P_{4r}}$$

and

$$T_{5r} = T_4$$

then, for reheating

$$T_5 = T_4\left\{1 - \eta_t\left[1 - \left(\frac{P_5}{P_4}\right)_e^{\frac{\gamma_t-1}{2\gamma_t}}\right]\right\}$$

and equation (B24) becomes

$$\frac{F}{\frac{W}{g}(1+f/a)} = \sqrt{C_v{}^2 2Jgc_{p,t}\left[1-\left(\frac{P_6}{p_0}\right)^{-\frac{\gamma_t-1}{\gamma_t}}\right]\left((1-\eta_x{}')T_4\left\{1-\eta_t\left[1-\left(K\frac{P_2}{p_0}\right)^{-\frac{\gamma_t-1}{2\gamma_t}}\left(\frac{P_6}{p_0}\right)^{\frac{\gamma_t-1}{2\gamma_t}}\right]\right\} + \eta_x{}'\,T_2\right) - \frac{V_0}{1+f/a} + \frac{2\eta_p\eta_t Jgc_{p,t}T_4}{V_0}\left[1-\left(K\frac{P_2}{p_0}\right)^{-\frac{\gamma_t-1}{2\gamma_t}}\left(\frac{P_6}{p_0}\right)^{\frac{\gamma_t-1}{2\gamma_t}}\right] - \frac{\eta_p Jgc_{p,c}\,\Delta T_c}{V_0(1+f/a)}} \tag{C1}$$

Differentiating equation (C1) with respect to $\left(\frac{p_6}{p_0}\right)^{\frac{\gamma_t-1}{\gamma_t}}$ and setting equal to zero gives

$$\frac{C_v^2 2Jgc_{p,t}\left((1-\eta_x')T_4\left\{1-\eta_t\left[1-\left(K\frac{p_2}{p_0}\right)^{-\frac{\gamma_t-1}{2\gamma_t}}\left(\frac{p_6}{p_0}\right)_{opt}^{\frac{\gamma_t-1}{2\gamma_t}}\right]\right\}+\eta_x'T_2\right)\left(\frac{p_6}{p_0}\right)_{opt}^{-\frac{3}{2}\frac{\gamma_t-1}{\gamma_t}}+C_v^2 2Jgc_{p,t}\left[1-\left(\frac{p_6}{p_0}\right)_{opt}^{-\frac{\gamma_t-1}{\gamma_t}}\right]\frac{1}{2}(1-\eta_x')T_4\,\eta_t\left(K\frac{p_2}{p_0}\right)^{-\frac{\gamma_t-1}{2\gamma_t}}}{2\sqrt{C_v^2 2Jgc_{p,t}\left[1-\left(\frac{p_6}{p_0}\right)_{opt}^{-\frac{\gamma_t-1}{\gamma_t}}\right]\left((1-\eta_x')T_4\left\{1-\eta_t\left[1-\left(K\frac{p_2}{p_0}\right)^{-\frac{\gamma_t-1}{2\gamma_t}}\left(\frac{p_6}{p_0}\right)_{opt}^{\frac{\gamma_t-1}{2\gamma_t}}\right]\right\}+\eta_x'T_2\right)}}=\frac{\eta_p\eta_t Jgc_{p,t}T_4}{V_0}\left(K\frac{p_2}{p_0}\right)^{-\frac{\gamma_t-1}{2\gamma_t}}\qquad(C2)$$

Then

$$\left[\frac{1}{2\left(\frac{p_6}{p_0}\right)_{opt}^{\frac{3}{2}\frac{\gamma_t-1}{\gamma_t}}}\right]\sqrt{\frac{(1-\eta_x{}')T_4\left\{1-\eta_t\left[1-\left(K\frac{p_2}{p_0}\right)^{-\frac{\gamma_t-1}{2\gamma_t}}\left(\frac{p_6}{p_0}\right)_{opt}^{\frac{\gamma_t-1}{2\gamma_t}}\right]\right\}+\eta_x{}'T_2}{1-\left(\frac{p_6}{p_0}\right)_{opt}^{-\frac{\gamma_t-1}{\gamma_t}}}}\ +$$

$$\left[\frac{1}{4}(1-\eta_x{}')T_4\ \eta_t\left(K\frac{p_2}{p_0}\right)^{-\frac{\gamma_t-1}{2\gamma_t}}\right]\sqrt{\frac{1-\left(\frac{p_6}{p_0}\right)_{opt}^{-\frac{\gamma_t-1}{\gamma_t}}}{(1-\eta_x{}')T_4\left\{1-\eta_t\left[1-\left(K\frac{p_2}{p_0}\right)^{-\frac{\gamma_t-1}{2\gamma_t}}\left(\frac{p_6}{p_0}\right)_{opt}^{\frac{\gamma_t-1}{2\gamma_t}}\right]\right\}+\eta_x{}'T_2}}$$

$$=\frac{\eta_p\eta_t Jgc_{p,t}T_4}{V_0\sqrt{C_v{}^2 2Jgc_{p,t}}}\left(K\frac{p_2}{p_0}\right)^{-\frac{\gamma_t-1}{2\gamma_t}}.$$

(C3)

and

$$\sqrt{2\left(K\frac{p_2}{p_0}\right)^{\frac{\gamma_t-1}{2\gamma_t}}\left(\frac{C_v}{\eta_p}\right)^2\frac{1-\eta_x{}'}{\eta_t}\frac{V_0{}^2}{Jgc_{p,t}T_4}\frac{\left(K\frac{p_2}{p_0}\right)^{\frac{\gamma_t-1}{2\gamma_t}}\left[\frac{1}{\eta_t}\left(1+\frac{\eta_x{}'}{1-\eta_x{}'}\frac{T_2}{T_4}\right)-1\right]+\left(\frac{p_6}{p_0}\right)_{opt}^{\frac{\gamma_t-1}{2\gamma_t}}}{\left(\frac{p_6}{p_0}\right)_{opt}^{3\frac{\gamma_t-1}{\gamma_t}}-\left(\frac{p_6}{p_0}\right)_{opt}^{2\frac{\gamma_t-1}{\gamma_t}}}}\;+$$

1361

$$\sqrt{\frac{1}{2}\left(K\frac{p_2}{p_0}\right)^{\frac{\gamma_t-1}{2\gamma_t}}\left(\frac{C_v}{\eta_p}\right)^2\frac{1-\eta_x{}'}{\eta_t}\frac{V_0{}^2}{Jgc_{p,t}T_4}\frac{1-\left(\frac{p_6}{p_0}\right)_{opt}^{-\frac{\gamma_t-1}{\gamma_t}}}{\left(K\frac{p_2}{p_0}\right)^{\frac{\gamma_t-1}{2\gamma_t}}\left[\frac{1}{\eta_t}\left(1+\frac{\eta_x{}'}{1-\eta_x{}'}\frac{T_2}{T_4}\right)-1\right]+\left(\frac{p_6}{p_0}\right)_{opt}^{\frac{\gamma_t-1}{2\gamma_t}}}}=2$$

(C4)

Letting

$$A=\left(\frac{p_6}{p_0}\right)_{opt}^{\frac{\gamma_t-1}{\gamma_t}}$$

$$B_r=\left(K\frac{p_2}{p_0}\right)^{\frac{\gamma_t-1}{2\gamma_t}}\left[\frac{1}{\eta_t}\left(1+\frac{\eta_x{}'}{1-\eta_x{}'}\frac{T_2}{T_4}\right)-1\right]$$

and

$$C_r = \left(K\frac{P_2}{p_0}\right)^{\frac{\gamma_t-1}{2\gamma_t}} \left(\frac{C_v}{\eta_p}\right)^2 \frac{1-\eta_x'}{\eta_t} \frac{V_0{}^2}{Jgc_{p,t}T_4}$$

equation (C4) may be written

$$\sqrt{\frac{2C_r(B_r+\sqrt{A})}{A^2(A-1)}} + \sqrt{\frac{C_r(A-1)}{2A(B_r+\sqrt{A})}} = 2 \tag{C5}$$

or

$$\sqrt{\frac{2C_r}{A^2(A-1)}}\left(\sqrt{B_r+\sqrt{A}}\right)^2 - 2\sqrt{B_r+\sqrt{A}} + \sqrt{\frac{C_r(A-1)}{2A}} = 0 \tag{C6}$$

When equation (C6) is solved for $\sqrt{B_r+\sqrt{A}}$

$$\sqrt{B_r+\sqrt{A}} = \frac{1\pm\sqrt{1-\sqrt{\frac{2C_r}{A^2(A-1)}}\sqrt{\frac{C_r(A-1)}{2A}}}}{\sqrt{\frac{2C_r}{A^2(A-1)}}} \tag{C7}$$

(Upon examination, the positive value was found to give the desired root.)

and

$$B_r = \frac{A^2(A-1)}{C_r}\left(1 - \frac{C_r}{2A^{3/2}} + \sqrt{1 - \frac{C_r}{A^{3/2}}}\right) - \sqrt{A} \tag{C8}$$

Equation (C8) is plotted in figure 2(b).

APPENDIX D

DISCUSSION OF ASSUMPTIONS

In the analysis for finding the optimum jet pressure ratio, the specific heat at constant pressure, c_p and the ratio of specific heats γ for both the turbine expansion and the jet expansion were assumed to be the same. For more accurate computation, especially when regeneration is used, it may be desirable to use average values of c_p and γ for the jet different from those used for the turbine. The use of different values of c_p and γ for turbine and jet does not have any appreciable effect upon the optimum jet pressure ratio $\left(\frac{P_6}{p_0}\right)_{opt}$, but may affect the total power output of the engine cycle. For this case, equation (B24) becomes

$$\frac{F}{\frac{W}{g}(1+f/a)} = \sqrt{c_v^{\,2}\,2Jgc_{p,j}\left[1-\left(\frac{P_6}{p_0}\right)^{-\frac{\gamma_j-1}{\gamma_j}}\right]\left((1-\eta_x')T_4\left\{1-\eta_t\left[1-\left(K\frac{P_2}{p_0}\right)^{-\frac{\gamma_t-1}{\gamma_t}}\left(\frac{P_6}{p_0}\right)^{\frac{\gamma_t-1}{\gamma_t}}\right]\right\}+\eta_x'\,T_2\right)} -$$

$$\frac{V_0}{1+f/a} + \frac{\eta_p\eta_t Jgc_{p,t}T_4}{V_0}\left[1-\left(K\frac{P_2}{p_0}\right)^{-\frac{\gamma_t-1}{\gamma_t}}\left(\frac{P_6}{p_0}\right)^{\frac{\gamma_t-1}{\gamma_t}}\right] - \frac{\eta_p Jgc_{p,c}\,\Delta T_c}{V_0(1+f/a)}$$

In the development of the equations for reheat (appendix C), it was assumed that the turbine pressure ratios before and after reheating were equal and therefore were equal to the square root of the effective turbine pressure ratio and that the reheat temperature was equal to the initial turbine-inlet temperature. In reference 2, it is proved that these conditions give maximum turbine power for an engine with two turbines with reheating. In the present analysis, the optimum jet pressure ratio did not vary appreciably as the reheat temperature was decreased from the turbine-inlet temperature if the pressure ratios in each turbine stage were held equal. If, however, the pressure ratio in the first turbine is either considerably increased or decreased from the square root of the over-all turbine

1361

pressure ratio while the reheat temperature is maintained equal to the turbine-inlet temperature, the optimum jet pressure ratio increases and approaches a value more nearly that for the optimum jet pressure ratio with no reheating.

5

REFERENCES

1. Kuhl, H.: Preliminary Report on the Fundamentals of the Control of Turbine-Propeller Jet Power Plants. NACA TM 1172, 1947.

2. English, Robert E., and Hauser, Cavour H.: A Method of Cycle Analysis for Aircraft Gas-Turbine Power Plants Driving Propellers. NACA TN 1497, 1948.

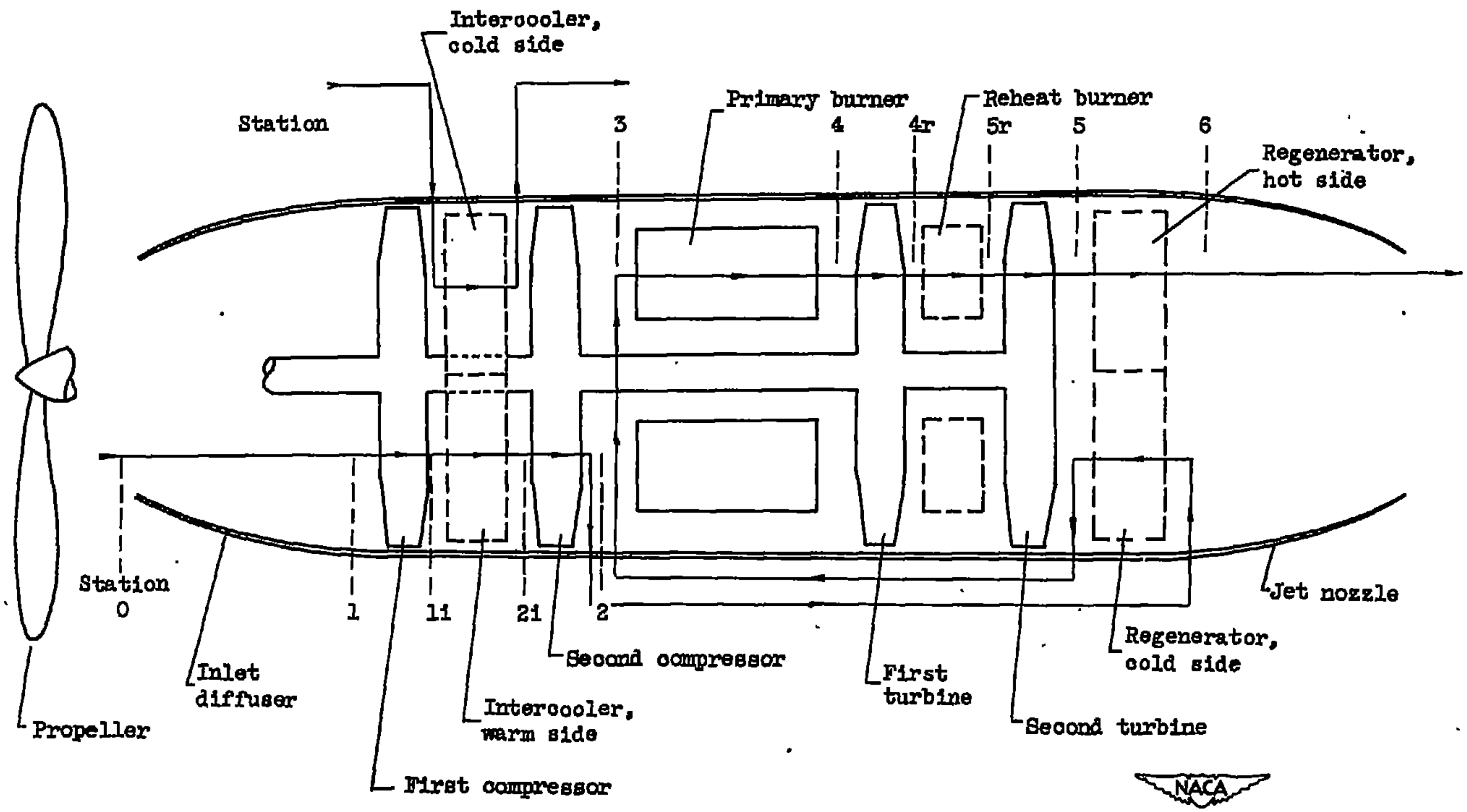
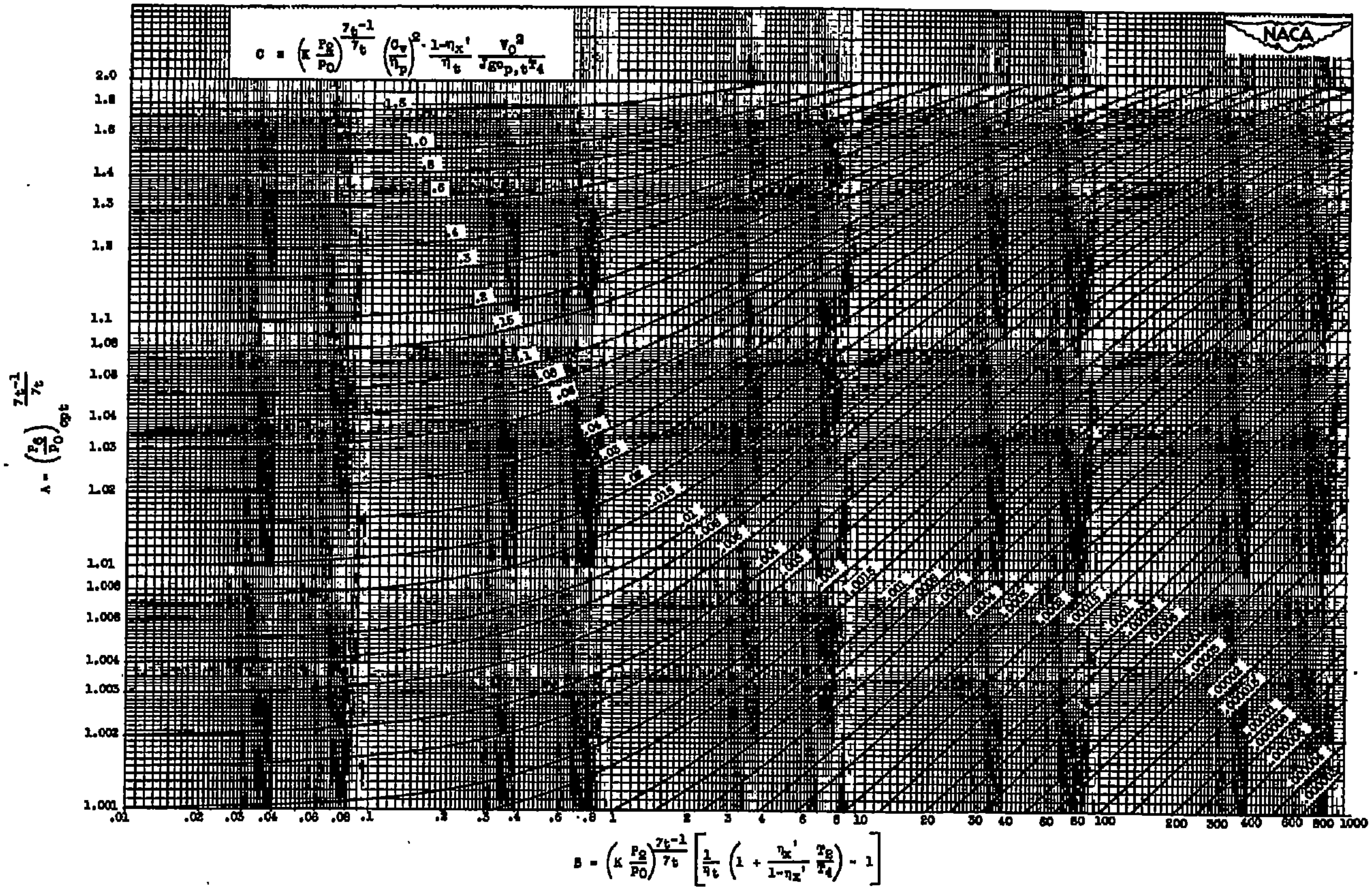
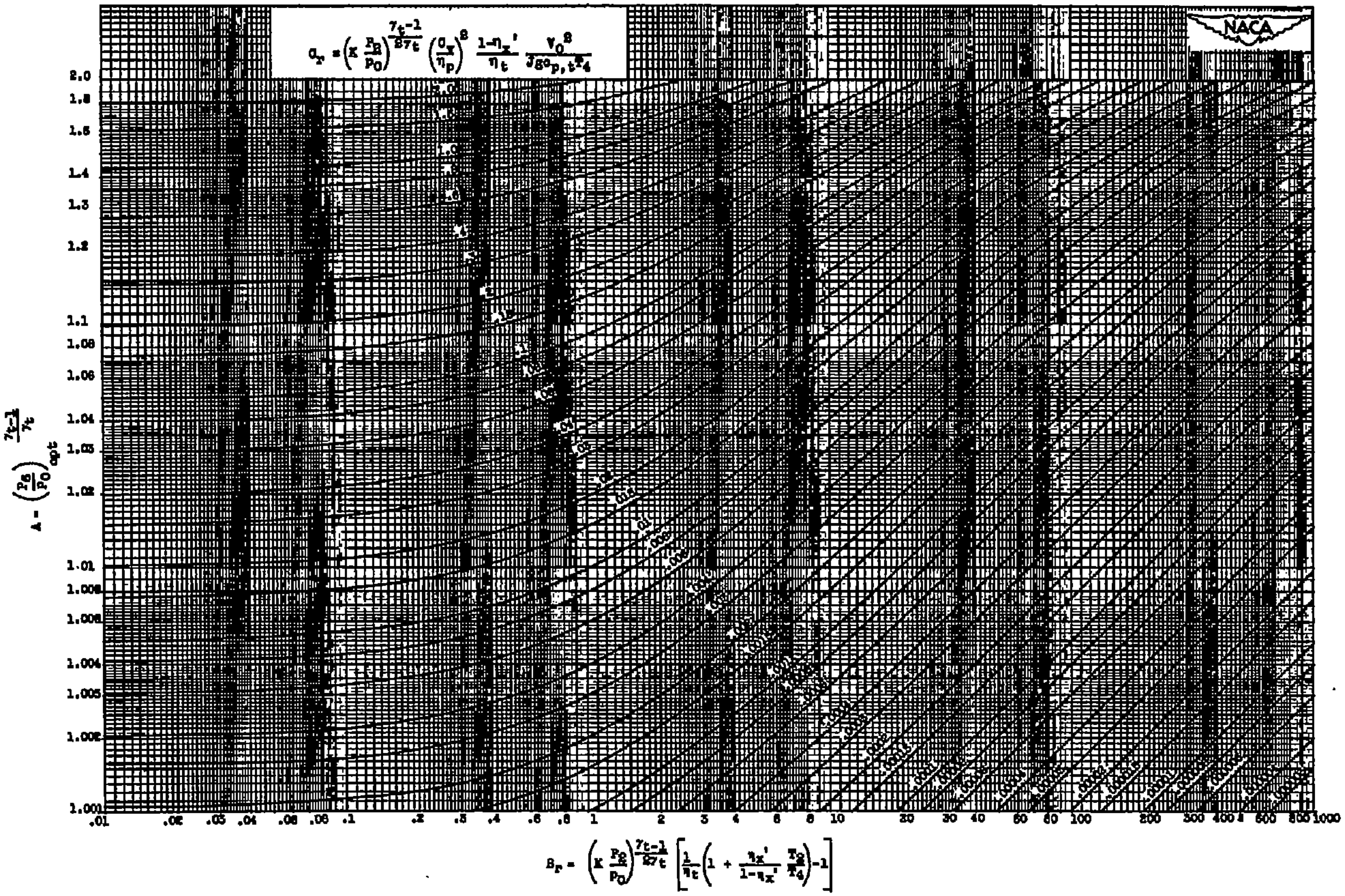


Figure 1. - Schematic diagram of turbine-propeller engine showing modifications.



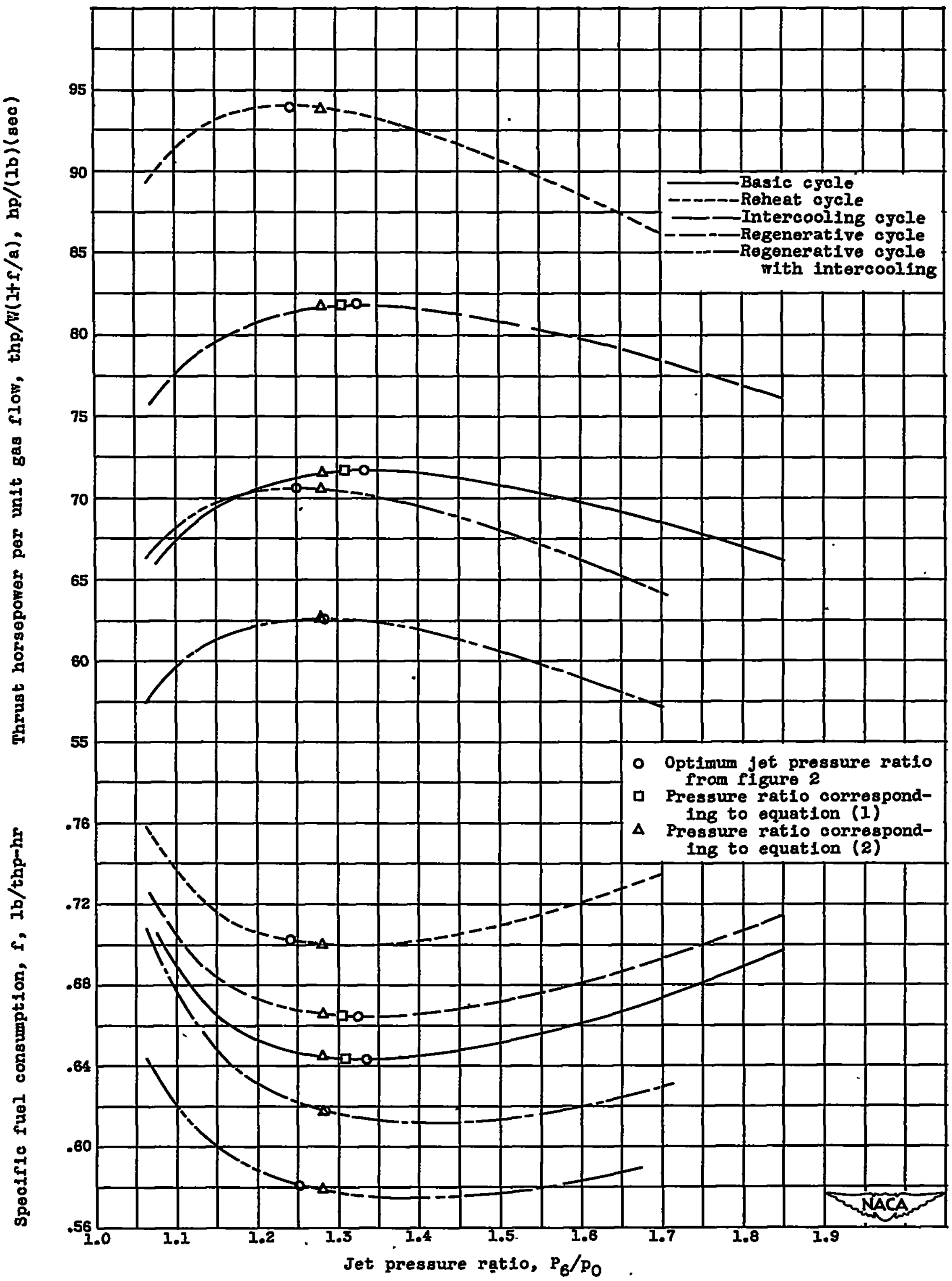
(a) Without reheat.

Figure 2. - Chart for determining optimum jet pressure ratio for engine with or without intercooling and regeneration.



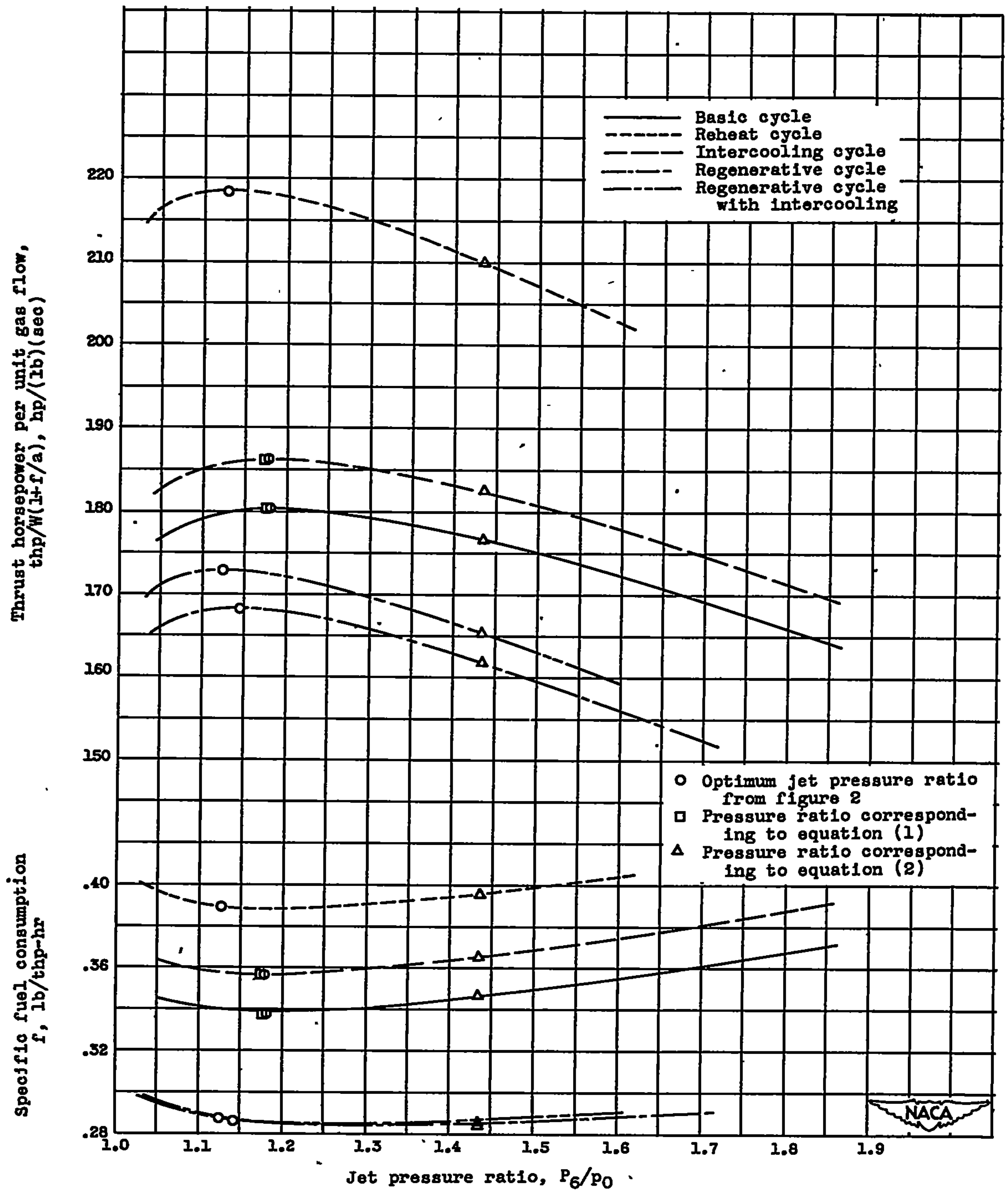
(b) With reheat.

Figure 2. - Concluded. Chart for determining optimum jet pressure ratio for engine with or without intercooling and regeneration.



(a) Condition I; moderate efficiencies and low altitude.

Figure 3. - Performance curves for various jet pressure ratios for turbine-propeller engine cycles at 500 miles per hour.

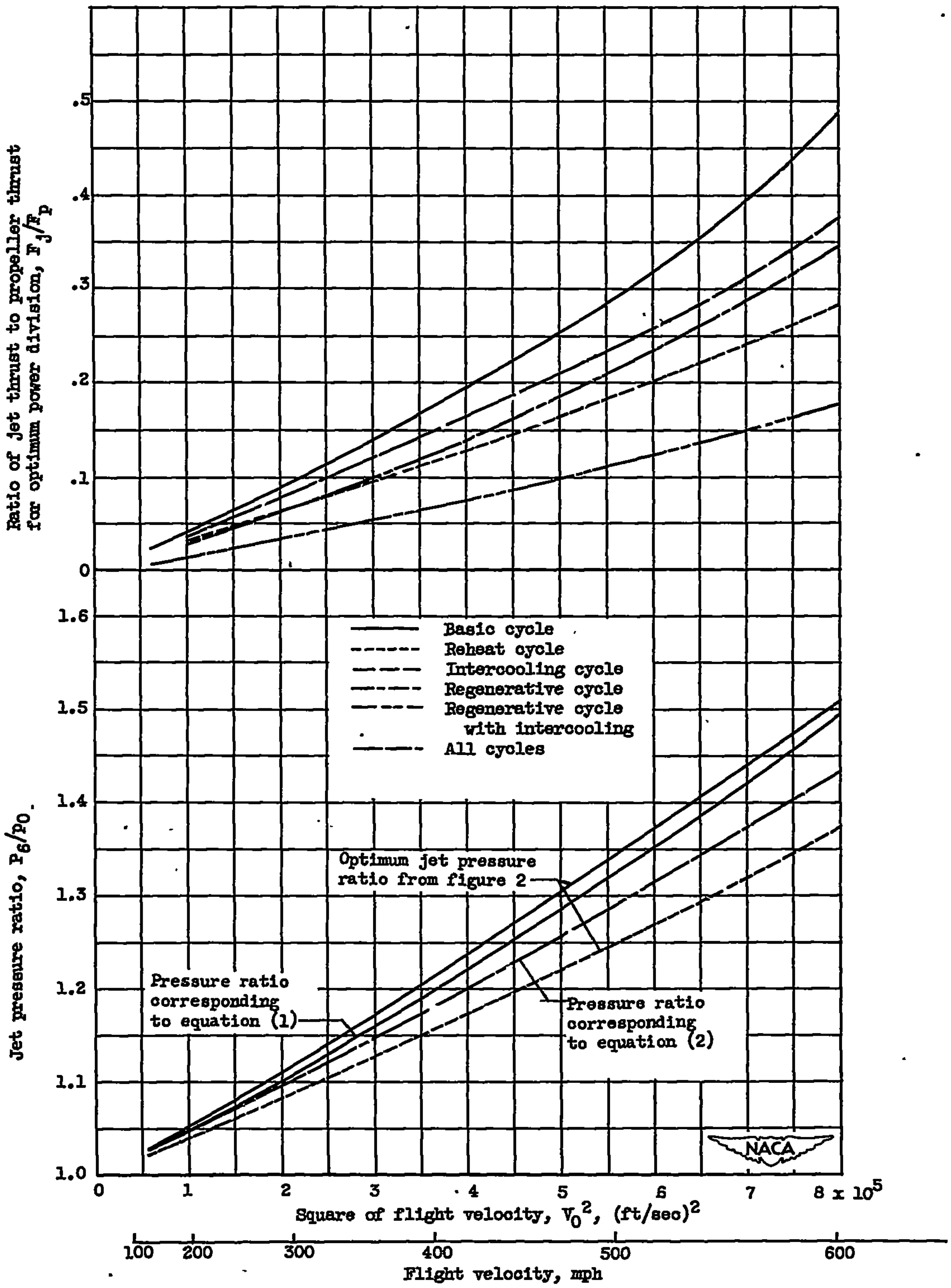


(b) Condition II; high efficiencies and high altitude.

Figure 3. - Concluded. Performance curves for various jet pressure ratios for turbine-propeller engine cycles at 500 miles per hour.

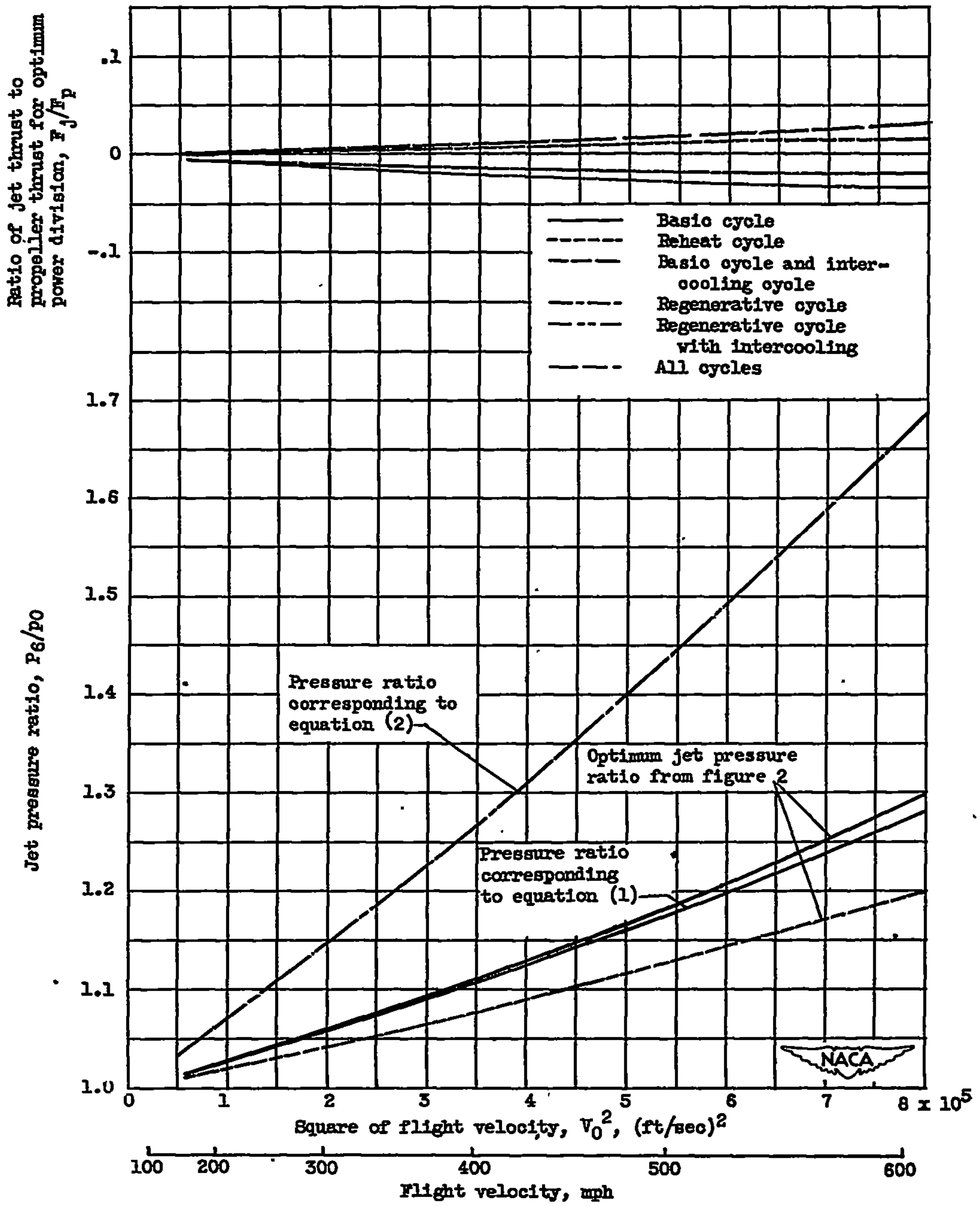
1321

1361



(a) Condition I; moderate efficiencies and low altitude.

Figure 4. - Ratio of jet thrust to propeller thrust and comparison of jet pressure ratios obtained by various methods for range of flight velocities for turbine-propeller-engine cycles.



(b) Condition II; high efficiencies and high altitude.

Figure 4. - Concluded. Ratio of jet thrust to propeller thrust and comparison of jet pressure ratios obtained by various methods for range of flight velocities for turbine-propeller-engine cycles.

NACA-Langley - 9-19-50 - 1050